STUDY GUIDE:

Module 10: Applying Constant Rates to Measurement

Measurements afford us a very subtle way to explore ratios. Namely, nouns take the place of fractional parts. For example, rather than talk about a 24th of a day, we talk about 1 hour; rather than talk about a 60th of an hour, we talk about 1 minute; and rather than talk about a 16th of a pound we talk about 1 ounce. One aim of this module is to apply our study of ratios to convert one measurement to another.

In early times it was convenient to choose conversion units that had many factors. For example, 12 was a convenient number in the sense that it had 2, 3, 4, and 6 as factors. But with the advent of place value, 10 became a very convenient conversion factor. Namely, with such conversion factors, decimal notation could be used very conveniently.

As an example of decimal conversions, we treat the metric system in this module and compare it with the English system that is based on such numbers as a dozen (12), a foot (12 inches), an hour (60 minutes).

In addition to studying the metric and the English systems in their own right we also show how we may use our previous studies to convert a measurement in one of these two systems into the other system.

Step 1:

View Videotape Lecture 10.

Step 2:

Read Module 10 of the text.

Step 3:

When you feel you understand the material presented in Steps 1 and 2, complete the following "Check-The-Main-Ideas" self-quiz by correctly filling in each of the blanks.

Check The Main Ideas:

There are 6 feet in 1 fathom. Hence	
in 5 fathoms there will be ___ X 6 or ____	*5; 30*
feet. This is an example of a _______	*constant*
rate. That is, no matter how many fathoms we	
have, there will always be _____ feet per	*6*
fathom. To convert fathoms to feet, we	
______ the number of fathoms by 6.	*multiply*
Conversely, to convert feet to fathoms,	
we take the number of feet and ______ by 6.	*divide*
For example, since $74 \div 6 = 12\frac{1}{3}$ there are	
______ fathoms in 74 feet.	*$12\frac{1}{3}$*
There is an intuitive way to decide whether	
we should multiply or divide by 6. Namely,	
since there are 6 feet per fathom, a fathom is	
_______ than a foot. That is, in a given length	*longer (greater)*
there are more ______ than ______. Since there	*feet; fathoms*
are more feet than fathoms in a given length, we	
convert from fathoms to feet by ______ by 6.	*multiplying*
Sometimes we avoid the use of fractions	
by using other denominations. For example,	
there are 12 inches in 1 foot and $\frac{2}{3}$ of 12 is ______.	*8*
Hence rather than say, say, $7\frac{2}{3}$ feet, we might	
say 7 feet, _____ inches. In a similar way	*8*
since there are 60 minutes in 1 hour and since	
$\frac{2}{5}$ of 60 = ______, then rather than write $3\frac{2}{5}$ hours	*24*
we might write 3 hours, ____ minutes.	*24*

After place value was invented, people chose	
powers of ______ as conversion factors. To	*ten*
indicate a thousand of a denomination, the prefix	
"_____" was used. Thus, a kilogram means _______	*kilo; 1,000*
grams. The prefix "milli" stands for "1 ______ of".	*thousandth (1,000th)*
Hence a thousandth of a gram would be called	
a ________. In other words, a kilogram is _______	*milligram; greater*
than a gram but a milligram is ______ than a gram.	*less*
So to convert grams to kilograms we move	
the decimal point 3 places to the _______. But	*left*
to convert grams to milligrams we move the decimal	
point 3 places to the _______.	*right*
The system that measures weight in grams is	
called the ______ system. In the ________ system	*metric; English*
weight is measured in pounds. Once we know that	
there are 454 grams in each pound, we can convert	
from pounds to grams by ________ by 454. That is,	*multiplying*
in a given weight there are _____ grams than pounds	*more*
because a pound is ______ than a gram. Once we know	*more*
the number of grams, we can convert to kilograms	
by _______ by 1,000.	*dividing*
We can write ratios as common fractions to	
convert from one unit to another. For example,	
to convert pounds to grams we want to _______	*divide*
by pounds and multiply by _______. That is:	*grams*

$$\text{pounds} \times \frac{\text{grams}}{\text{pounds}} = \underline{\qquad\qquad}$$

So to convert 30 pounds to grams,

we'd write:

$$\frac{30 \text{ pounds}}{1} \times \frac{454 \text{ grams}}{1 \text{ pound}} = (30 \times \underline{\quad\quad}) \text{ grams} \qquad\qquad \textit{454}$$

$$= 13{,}620 \text{ grams}$$

Had we wanted to convert from grams to pounds,

then we'd have divided by ______ and multiplied — *grams*

by ________. Whether we multiply by pounds or — *pounds*

divide by pounds, there are still ______ grams — *454*

per pound. That is, whether grams is in the

numerator or denominator, it is still multiplied

by _____. — *454*

We must keep in mind that the methods we are

discussing in this module apply to all _______ — *constant*

rates. For example if we know that there are

4 "moogs" per "goog" and we want to convert

from "moogs" to "googs" we divide by ______ — *"moogs"*

and multiply by ________. That is: — *"googs"*

$$\frac{\cancel{\text{moogs}}}{1} \times \frac{\text{googs}}{\cancel{\text{moogs}}} = \text{googs}$$

In particular, then, since "moogs" are in the

denominator, we convert from "moogs" to "googs"

by _______ by 4. *The key point is that we don't* — *dividing*

even have to know what "moogs" and "googs" are to

solve this conversion problem.

****

Step 4:

Do the Mastery Review on the next page.

Mastery Review:

ANSWERS

1. If there are 110 fathoms per furlong, how many fathoms are there in 8 furlongs? 1. ____________

2. How many furlongs are there in 990 fathoms? 2. ____________

3. How many feet are there in 8 yards? 3. ____________

4. A fathom is 6 feet and a furlong is $\frac{1}{8}$ of a mile. How many feet are there in 2 furlongs? 4. ____________

5. How many deciliters are there in 1 liter? 5. ____________

6. How many decidollars are there in 1 dollar? 6. ____________

7. How many centiliters in 1 liter? 7. ____________

8. How many centidollars are there in 1 dollar? 8. ____________

9. How many liters are there in a kiloliter? 9. ____________

10. How many dollars are there in a kilodollar? 10. ____________

11. How many liters are there in a milliliter? 11. ____________

12. If there are 2.54 centimeters in 1 inch, how many millimeters are there in 2 inches? 12. ____________

13. To the nearest hundredth, how many inches are there per centimeter? 13. ____________

14. Which is longer, 600 centimeters or 20 feet? 14. ____________

15. At $2.54 per pound, how much would 10 pounds of ground beef cost? 15. ____________

16. If there are 2.54 centimeters per inch, how many centimeters are there in 10 inches? 16. ____________

17. There are 454 grams per pound. How many grams are there in 20 pounds? 17. ____________

18. How many kilograms are there in 20 pounds? 18. ____________

19. How many pounds are there in 20 kilograms? Write the answer as a decimal fraction, rounded off to the nearest hundredth of a pound? 19. ____________

(cont)

Mastery Review: (cont)

ANSWERS:

20. Convert 0.55 hours into minutes. 20. ____________

21. Convert 0.9 pounds to the nearest ounce. 21. ____________

22. Convert 0.9 feet to the nearest tenth of an inch. 22. ____________

23. Convert 0.9 centimeters to millimeters. 23. ____________

24. Convert 45.67 centimeters to millimeters. 24. ____________

25. Convert 45.67 millimeters to centimeters. 25. ____________

26. Convert 4.237 kilometers to meters. 26. ____________

27. Convert 4.237 kilometers to millimeters. 27. ____________

28. Convert 987 millimeters to kilometers. 28. ____________

29. A hectare is 2.471 acres. How many acres are there in 3,000 hectares? 29. ____________

30. To the nearest hectare, how many hectares are there in 3,000 acres? 30. ____________

Answers:

1. 880 2. 9 3. 24 4. 1,320 5. 10

6. 10 7. 100 8. 100 9. 1,000 10. 1,000

11. 1/1,000 (0.001) 12. 50.8 13. 0.39 14. 20 feet

15. $25.40 16. 25.4 17. 9,080 18. 9.08

19. 44.05 20. 33 minutes 21. 14 ounces 22. 11 inches

23. 9 mm 24. 456.7 mm 25. 4.567 cm 26. 4,237 m

27. 4,237,000 mm 28. 28. 0.000987 km

29. 7,413 30. 1,214

Step 5:

Do Self-Test 10, Form A on the next page.

ANSWERS

Unless other conversion factors are given, use only those conversions given in Module 10.

1. To the nearest foot, how many yards are there in 100 meters? — 1. ______

2. To the nearest centimeter, how many meters are there in 100 yards? — 2. ______

3. At a cost of $350 per hectare, how much will 55 acres cost? Round off your answer to the nearest dollar. — 3. ______

4. At a cost of $350 per acre, to the nearest dollar, what is the cost of 55 hectares? — 4. ______

5. At a rate of $0.0073 per gram, what is the cost, to the nearest cent, of: — 5. (a) ______ (b) ______

   (a) 1 pound?

   (b) 1 metric ton?

6. At a rate of 40 miles per hour, how far will you travel in 3 hours and 14 minutes? Write the answer to the nearest tenth of a mile. — 6. ______

7. Access time to a computer costs $288 an hour where the time is measured to the nearest second. At this rate what will it cost if you use the computer for 3 hours, 12 minutes, 42 seconds? — 7. ______

8. A tablespoon is 2 ounces and a cup is 8 ounces. A recipe calls for 2 1/2 cups of milk. You have a tablespoon but no measuring cup. How many tablespoons of milk should you use for this recipe? — 8. ______

9. 48 kilograms of tobacco are packed into 1-pound bags. — 9. (a) ______ (b) ______

   (a) How many 1-pound bags are filled by the tobacco?

   (b) To the nearest ounce, how many ounces of tobacco are left after the bags are filled?

10. There are 7,000 grains in one pound. If a "pinch" of salt is 20 grains; to the nearest whole number, how many pinches are there in a 1 pound, 10 ounce box of salt? — 10. ______

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 10, Form A

1. *109 yards. 1 foot (109.36133 yards)*
2. *91.44 meters (91 meters, 44 centimeters)*
3. *$7,790*
4. *$47,567*
5. *(a) $3.31 (b) $7,300*
6. *129.3 miles*
7. *$924.96*
8. *10*
9. *(a) 105 (b) 12*
10. *569*

****

If you did each problem in Self-Test 10, Form A correctly, you may, if you wish, proceed to the next module. Otherwise, continue with Step 6.

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Step 6:

Study the solutions for Self-Test 10, Form A, giving special attention to any problems you failed to solve correctly.

Solutions for Self-Test 10, Form A

1.

In the module we showed how to use the common fraction notation to convert from one unit to another. In this exercise, we use that principle several times in succession. For example, if want to convert 100 meters to centimeters, we'd divide by meters and multiply by centimeters. That is:

$$100 \text{ m} = \frac{100 \text{ m}}{1} \times \frac{100 \text{ cm}}{1 \text{ m}}$$

$$= \frac{100 \cancel{\text{m}}}{1} \times \frac{100 \text{ cm}}{1 \cancel{\text{m}}}$$

$$= 10{,}000 \text{ cm} \qquad (1)$$

We then use our cross-over relationship of 2.54 centimeters per inch to convert centimeters to inches. Namely, we'll divide by centimeters and multiply by inches to get:

$$10{,}000 \text{ cm} = \frac{10{,}000 \text{ cm}}{1} \times \frac{1 \text{ in}}{2.54 \text{ cm}}$$

$$= \frac{10{,}000 \cancel{\text{cm}}}{1} \times \frac{1 \text{ in}}{2.54 \cancel{\text{cm}}}$$

$$= \frac{10{,}000}{2.54} \text{ inches}$$

$$\doteq 3{,}937 \text{ inches} \qquad (2)$$

Now we use the fact that there are 12 inches per foot to convert inches to feet. That is, starting with (2) we divide by inches and multiply by feet to get:

$$3{,}937 \text{ inches} = \frac{3{,}937 \cancel{\text{in}}}{1} \times \frac{1 \text{ ft}}{12 \cancel{\text{in}}}$$

$$= \frac{3{,}937}{12} \text{ feet}$$

Based on the material of this module 2.54 cm per inch will be our cross-over conversion. The sequence of conversions will be:

meters
↓
centimeters
↓
inches
↓
feet
↓
yds

This step requires that you know there are 100 cm/m.

(1) would be the right answer if the exercise asked us to find the number of centimeters in 100 meters.

Whether we talk about cm/in or in/cm the 2.54 modifies cm. Since cm is in the denominator, so is 2.54.

My calculator gives 3937.0079 as the quotient, but since 2.54 is already a rounded off number, there is no need to carry more than 4 digits in our answer. So we've shown that there are 3,937 inches in 100 meters.

Based on our above approach, we've combined two steps here.

1. (cont)

Hence to the nearest foot, there are 328 feet in 100 meters. We now use the fact that there are 3 feet per yard to get:

My calculator computes the quotient to be 328.08333 (which means 328 feet 1 inch)

$$
\begin{aligned}
328 \text{ feet} &= \frac{328 \text{ ft}}{1} \times \frac{1 \text{ yd}}{3 \text{ ft}} \\
&= \frac{328}{3} \text{ yd} \\
&= 109.\overline{3} \text{ yards} \\
&\doteq 109 \text{ yards}
\end{aligned}
$$

While the process was not difficult, it was tedious. To avoid the tedium we combine the steps into one process. Namely, we use common fractions one factor at a time to convert from meters to centimeters to inches to feet and then to yards as follows:

$$
\begin{aligned}
100 \text{ m} &= \frac{100 \text{ m}}{1} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{1 \text{ in}}{2.54 \text{ cm}} \times \frac{1 \text{ ft}}{12 \text{ in}} \times \frac{1 \text{ yd}}{3 \text{ ft}} \\
&= \frac{100 \times 100 \times 1 \times 1 \times 1}{1 \times 1 \times 2.54 \times 12 \times 3} \text{ yards} \\
&= \frac{10{,}000}{91.44} \text{ yards}
\end{aligned}
$$

Just as with numbers, any "label" in a numerator can be cancelled with that same "label" in the denominator.

$$
\begin{array}{rl}
 & \ \ 109.36\ldots\ \text{yards} \\
= 9{,}144 & \hspace{-0.5em}\overline{)1{,}000{,}000.00\ldots\ldots} \\
 & -\ 914\ 4 \\
 & \quad 85\ 60 \\
 & -\ 00\ 00 \\
 & \quad 85\ 600 \\
 & -\ 82\ 296 \\
 & \quad\ 3\ 3040 \\
 & -\ 2\ 7432 \\
 & \quad\ \ 56080 \\
 & -\ 54864 \\
 & \quad\ \ \ 1216
\end{array}
$$

Another advantage of this method is that we can avoid having to round off until the last step. This preserves accuracy.

These steps are supplied to help encourage you to use the calculator. Namely, enter 10,000, press the division key, enter 91.44 and press the equal key. At once you get: 109.36133

1. (concluded)

Remember that 109.36 or $109\frac{36}{100}$ yards means 109 yards + $\frac{36}{100}$ yards. But $\frac{36}{100}$ of a yard is the same as $\frac{36}{100}$ of 3 feet or $\frac{108}{100}$ or 1.08 feet. This rounds off to 1 foot.

The decimal fraction part is the portion of a yard that's the remainder. Since there are 3 feet per yard, we take the decimal part of *3* *feet when we convert to feet.*

Hence, to the nearest foot:

100 meters = 109 yards, 1 foot

Note:

We don't need to use the common fraction notation to solve this problem. Remembering to multiply when we convert to a smaller unit and to divide when we convert to a greater unit, we have:

My calculator's display shows the following numbers:

(1) Start with the number of meters (100)	*100 (meters)*
(2) Multiply by 100 to get centimeters.	*10,000 (centimeters)*
(3) Divide by 2.54 to get inches.	*3,937.0079 (inches)*
(4) Divide by 12 to get feet.	*328.08399 (feet)*
(5) Divide by 3 to get yards.	*109.36133 (yards)*

The point is that unless you have a good intuitive sense about what's happening, it's easy to multiply when you should have divided etc. By taking the time to use common fractions one step at a time, you reduce considerably the chances for making a serious error.

Remember that 0.36133 yards means:
0.36133 X 1 yard =
0.36133 X 3 feet =
1.08399 feet $\doteq$
1 foot

While the situations in other exercises are different, the idea we use in this exercise is the same one we use in the rest. So don't get "hung up" on the metric system. It is simply one of an endless number of applications of constant rates.

2.

This time we want to start with yards and convert to meters. If we proceed one step at a time, we go successively from yards to feet, from feet to inches, from inches to centimeters, and from centimeters to meters. In essence this reverses the procedure we used in Exercise 1. Namely:

(1) Start with the number of yards (100)
(2) Multiply by 3 to get feet.
(3) Multiply by 12 to get inches.
(4) Multiply by 2.54 to get centimeters
(5) Divide by 100 to get meters.

On my calculator:

100 (yards)
300 (feet)
3,600 (inches)
9,144 (centimeters)
91.44 (meters)

The answer is 91.44 meters, which, because of the decimal nature of the metric system, is the same as 91 meters 44 centimeters.

See the advantage of the metric system? 91.44 feet is not 91 feet 44 inches.

If you try to memorize the conversion factors without understanding, it is easy to confuse the steps in Exercise 1 with those in Exercise 2. Notice how using the common fraction notation can be helpful:

$$100 \text{ yds} = \frac{100 \cancel{\text{yds}}}{1} \times \frac{3 \cancel{\text{ft}}}{1 \cancel{\text{yd}}} \times \frac{12 \cancel{\text{in}}}{1 \cancel{\text{ft}}} \times \frac{2.54 \cancel{\text{cm}}}{1 \cancel{\text{in}}} \times \frac{1 \text{ m}}{100 \cancel{\text{cm}}}$$

$$= \frac{100 \times 3 \times 12 \times 2.54 \times 1}{1 \times 1 \times 1 \times 1 \times 100} \text{ meters}$$

$$= \frac{9,144}{100} \text{ meters}$$

$$= 91.44 \text{ meters}$$

91.44 meters names the same length as a 100 yards. So a meter is a little more than a yard (about 10%) because it takes few of them (91.44) to measure the same length than it takes yards (100).

So if you have to deal with meters, think of them as being a bit more than a yard.

3.

Up to now we've been dealing with situations of the type:

$$\frac{\text{dollars}}{\text{hectare}} \text{ X hectares = dollars} \quad (1)$$

or

$$\frac{\text{dollars}}{\text{acre}} \text{ X acres = dollars} \quad (2)$$

What makes Exercise 3 a bit different from we've done before is that we have a mixture of equations (1) and (2) here. That is, we have "dollars per hectare" as our rate and "acres" as our unit item. In other words, in common fraction form we'd have:

We can cancel "acres" with "acres" and "hectares" with "hectares", but we can't cancel "acres" with "hectares

$$\frac{350 \text{ dollars}}{1 \text{ hectare}} \text{ X 55 acres} \quad (3)$$

and this creates problems because there is no common "label" to cancel.

The strategy in this case is to either convert acres to hectares or hectares to acres in (3).

So one way or another we're going to have recall that there are 2.471 acres per hectare.

By our previous methods we have:

$$55 \text{ acres} = \frac{55 \text{ acres}}{1} \text{ X } \frac{1 \text{ hectare}}{2.471 \text{ acres}}$$

$$= \frac{55}{2.471} \text{ hectares} \quad (4)$$

In this form, the "acres" cancel and leave us with "hectares" as the label.

We can replace 55 acres in (3) by its value in (4) to get:

$$\frac{350 \text{ dollars}}{1 \text{ hectare}} \text{ X } \frac{55}{2.471} \text{ hectares} =$$

$$\frac{350 \text{ X } 55}{2.471} \text{ dollars} \doteq$$

$7,790

where we've rounded off to the nearest dollar.

The "hectares" cancel and the label becomes "dollars".

On the calculator:
1. *Enter 350*
2. *Multiply by 55*
3. *Divide by 2.471*

3. (cont)

Again notice that we could have solved this problem without using the common fraction notation. Namely the fact that there are 2.471 acres per hectare tells us that the hectare is a greater unit than the acre. Hence:

(1) Start with the price per hectare ($350)

(2) Divide by 2.471 to find the price per acre.

(3) Multiply by 55 to find the price of 55 acres.

Remember that since a hectare is more than an acre we have to divide by 2.471 to find the cost per acre.

This procedure is more direct and involves fewer steps than the method of common fraction notation, but the common fraction notation gives us "clues" as to when we multiply and when we divide.

Notice that in the previous method, we first multiplied by 55 and then divided by 2.471 but either way we get the same asnwer.

Note 1:

In terms of fill-in-the-blank we could have written:

$$55 \text{ acres} = \underline{\qquad\quad} \text{ dollars}$$

We'd then have used the sequence of conversions:

$$55 \text{ acres} \times \frac{1 \text{ hectare}}{2.471 \text{ acres}} \times \frac{350 \text{ dollars}}{1 \text{ hectare}} = \frac{55 \times 350}{2.471} \text{ dollars.}$$

Both "acres" and "hectares" cancel, leaving us with "dollars" as the label.

Note 2:

Don't be afraid to estimate. For example, since 2.471 is about 2.5, a hectare is about $2\frac{1}{2}$ acres. That is there are about 5 acres per 2 hectares. Hence since $55 \div 5 = 11$, there are about 11×2 or 22 hactares in 55 acres. At

$$\frac{2.5 \text{ acres}}{1 \text{ hectare}} = \frac{2.5 \times 2 \text{ acres}}{1 \times 2 \text{ hectares}}$$

Solutions for Self-Test 10, Form A (cont)

3. (concluded)

a rate of $350 per hectare, 22 hectares will cost about 22 X $350. Since 2 X 35 = 70, 20 X 350 = 7,000. Hence we know that the answer has to be in excess of $7,000. So $7,790 is a plausible answer.

4.

This is the "inverse" of the previous exercise. This time our sequence of steps is:

(1) Start with the cost per acre ($350)

(2) Multiply by 2.471 to get the cost per hectare.

(3) Multiply by 55 to find the cost of 55 hectares.

Since a hectare is more tha an acre, a hectare costs more than an acre. That's why we multiply in (2).

Using a calculator and rounding off to the nearest dollar, we get $47,567.

Do you see how tedious it would be long-hand to multi ply 350 by 2.471 and then t divide this product by 2.471

If you use the common fraction method you get:

$$\frac{55 \text{ hectares}}{1} \times \frac{2.471 \text{ acres}}{1 \text{ hectare}} \times \frac{350 \text{ dollars}}{1 \text{ acre}} =$$

$$\frac{55 \times 2.471 \times 350}{1 \times 1 \times 1} \text{ dollars} \doteq$$

47,567 dollars

What we're doing here is filling in the blank in:

55 hectares = ______ dollar

In the common fraction form "hectares" and "acres" canc and we're left with "dollar as the label.

Use whatever method you feel most comfortable with but be careful to distinguish Exercise 3 from Exercise 4.

Remember; we can't get away from reading comprehension no matter how many technolo ical aids we have.

5.

In both (a) and (b) we deal with conversions. In (a) we want to convert from "dollars per gram" to "dollars per pound" and in (b) we want to convert from "dollars per gram" to "dollars per metric ton".

Computationally (b) is simpler than (a) because we stay within the metric system where we use only powers of ten.

While this may look different from Exercises 3 and 4, it is essentially the same format applied to a different application.

(a)

In terms of fill in the blank we have:

1 pound = ______ dollars

We can then use the chain of steps:

$$\frac{1\text{ pound}}{1} \times \frac{454\text{ grams}}{1\text{ pound}} \times \frac{0.0073\text{ dollars}}{1\text{ gram}} =$$

$$\frac{454 \times 0.0073}{1}\text{ dollars} =$$

$$3.3142\text{ dollars} \doteq$$

$$\$3.31$$

We're first converting pounds to grams, and then grams to dollars using the constant rate of $0.0073 per gram.

Remember that we're being asked to round off to the nearest cent.

Without using the common fraction notation, we may proceed as follows:

(1) Start with $0.0073 (the cost per gram)

(2) Multiply by 454 to find the cost per pound.

All you have to know here is that there are 454 grams per pound. That is, a pound costs 454 times as much as a gram.

Note:

Both $0.0073 and $0.007 look like "scrawny" amounts. But per pound $0.0073 per grams amounts to $3.31; and if we had rounded $0.0073 off to $0.007 the cost per pound would have been $0.007 X 454 or $3.18 which is quite different from the correct answer, $3.31.

Even more dramatically, if we round off to $0.01, we get $0.01 X 454 or $4.54 per pound Small errors can mount quickly when we deal with large amounts.

5(b)

This time we have to know that there are 1,000 grams per kilogram and 1,000 kilograms per metric ton. In terms of money:

1,000 X cost per gram = cost per kilogram

1,000 X cost per kilogram = cost per metric ton

In other words,

$$\frac{\$0.0073}{1 \text{ gram}} = \frac{\$0.0073 \text{ X } 1000}{1 \text{ X } 1000 \text{ grams}}$$

$$= \frac{\$7.30}{1 \text{ kg}}$$

$$= \frac{\$7.30 \text{ X } 1,000}{1 \text{ X } 1000 \text{ kg}}$$

$$= \frac{\$7,300}{1 \text{ metric ton}}$$

In terms of common fractions

$$\frac{\$0.0073}{1\ \cancel{gm}} \text{ X } \frac{1000\ \cancel{gm}}{1\ \cancel{km}} \text{ X } \frac{1000\ \cancel{km}}{1 \text{ m ton}} =$$

$$\frac{\$.0073 \text{ X } 1,000 \text{ X } 1,000}{1 \text{ metric ton}}$$

Notice that if we round \$0.0073 to \$0.007, we get an error of \$300 per metric ton (that is, \$7,000 rather than \$7,300)

6.

Here we again have an example of the same principle but in a different environment. The constant rate idea here is:

$$\frac{\text{miles}}{\text{hour}} \text{ X hours = miles} \qquad (1)$$

The only problem is that the time is given in terms of both hours and minutes. The thing to remember here is that 3 hours and 14 minutes is another way of saying $3\frac{14}{60}$ or $3\frac{7}{30}$ hours.

That is, 1 minute = $\frac{1}{60}$ *hours*

Since the constant speed is 60 miles per hour, we use (1) in the form:

$$\frac{40 \text{ miles}}{1 \text{ hour}} \text{ X } 3\frac{7}{30} \text{ hours} =$$

$$40 \text{ X } 3\frac{7}{30} \text{ miles} =$$

The "hours" cancel, leaving us with "miles". This principle doesn't change just because we're dealing with mixed numbers.

6. (cont)

$$= 40 \text{ X } (3 + \frac{7}{30}) \text{ miles}$$

$$= (40 \text{ X } 3) + (40 \text{ X } \frac{7}{30}) \text{ miles}$$

We're using the distributive property here.

$$= 120 \text{ miles} + \frac{280}{30} \text{ miles}$$

$$= 120 \text{ miles} + 9\frac{1}{3} \text{ miles}$$

$$= 129\frac{1}{3} \text{ miles}$$

$$\doteq 129.3 \text{ miles}$$

Note 1:

If we prefer common fractions to mixed numbers remember that $3\frac{7}{30}$ can be rewritten as $\frac{97}{30}$. In this form we'd get:

$$40 \text{ X } \frac{97}{30} = \frac{40 \text{ X } 97}{30}$$

$$= \frac{4 \text{ X } 97}{3}$$

$$= \frac{388}{3}$$

$$= 129\frac{1}{3}$$

If you did this step on a calculator, you'd get the answer in the form:
129.33333
which rounds off to 129.3

Note 2:

40 miles per hour is the same as 40 miles per 60 minutes or $\frac{40}{60}$ (= $\frac{2}{3}$) miles per minute.

3 hours and 14 minutes =

3 X 60 minutes + 14 minutes =

180 minutes + 14 minutes, or 194 minutes.

We can then use:

$$\frac{2 \text{ miles}}{3 \text{ minutes}} \text{ X } 194 \text{ minutes} =$$

$$\frac{2 \text{ X } 194}{3} \text{ miles} = \frac{388}{3} \text{miles, which}$$

agrees with our previous answer.

In short, it's up to use whether to use:

$$\frac{\text{miles}}{\text{hours}} \text{ X hours = miles}$$

or

$$\frac{\text{miles}}{\text{minute}} \text{ X minutes = miles}$$

The theory is the same either way but the conversion factors are different.

6. (concluded)

As a final note, again notice the value of being able to estimate. 14 minutes is just a little less than 15 minutes, which is $\frac{1}{4}$ hour. If you can go 40 miles in 1 hour, then at that rate, you'll go 10 miles in $\frac{1}{4}$ hours. So:

$$\begin{array}{rl} 3 \text{ hrs} & @\ 40 \text{ mph} = 120 \text{ miles} \\ \frac{1}{4} \text{ hr} & @\ 40 \text{ mph} = 10 \text{ miles} \\ \hline 3\frac{1}{4} \text{ hr} & @\ 40 \text{ mph} = 130 \text{ miles} \end{array}$$

So we know that the correct answer should be a little less than 130 miles; and 129 miles is certainly consistent with this fact.

7.

Again we have a different application of the same principle. There are many forms we could use here, one of which is:

$$\frac{\text{dollars}}{\text{second}} \text{ X seconds} = \text{dollars} \qquad (1)$$

We could also use:

$$\frac{dollars}{hour} \; X \; hours = dollars$$

If we express hours in terms of minutes and/or seconds, the conversion factors are whole numbers; but if we express minutes and/or seconds in terms of hours the conversion factors will be fractions.

We're told that the cost is \$288 per hour.

Well:

$$\frac{\$288}{1 \text{ hr}} = \frac{\$288}{60 \text{ min}} = \frac{\$4.80}{1 \text{ min}}$$

$$\frac{\$4.80}{1 \text{ min}} = \frac{\$4.80}{60 \text{ sec}} = \frac{\$0.08}{1 \text{ sec}}$$

That is: \$288 per 1 hour =

\$288 per 60 minutes =

\$4.80 per 1 minute =

\$4.80 per 60 seconds =

\$0.08 per 1 second

Mathematically, this is the same thing that we did in th[e] common fraction form; but psychologically, one way ma[y] seem easier than the other.

7. (cont)

We then convert 3 hours, 12 minutes, and 42 seconds into seconds. Namely:

3 hours = 3 X 60 minutes

= 180 minutes

= 180 X 60 seconds

= 10,800 seconds

12 minutes = 12 X 60 seconds

= 720 seconds

Hence: 3 hours 12 minutes 42 seconds =

(10,800 + 720 + 42) seconds =

11,562 seconds

Now we return to (1) and write:

$\frac{0.08 \text{ dollars}}{1 \text{ second}}$ X 11,562 seconds =

0.08 X 11,562 dollars =

$924.96

Rough Check:

3 hours @ $288 = $ 864

4 hours @ $288 = $1,152

3 hours 12 minutes 42 seconds is between 3 hours and 4 hours but closer to 3 hours. Hence the cost should be between $864 and $1,152 but closer to $864.

Notice that we could have used the common fraction format to see how much 11,562 seconds would cost. We'd convert from seconds to minutes to hours to dollars:

We could have said that 12 minutes is 12/60 or 1/5 or 0.2 hours and that 42 seconds is 42/60 or 7/10 or 0.7 minutes or 0.7/60 or $0.01\overline{16}$ hours. So in hours, we have $3 + 0.2 + 0.1\overline{16} \doteq 3.31\overline{6}$ hours. We then multiply this by $288 to get: $924.96. But I feel it's easier to use seconds than hours in this exercise.

The "seconds" cancel and we are left with "dollars"

Even with a calculator it's a good idea to use this type of rough check. It at least ensures that you're answer is in the proper range.

7. (concluded)

$$\frac{11{,}562 \text{ sec}}{1} \times \frac{1 \text{ min}}{60 \text{ sec}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{288 \text{ dollars}}{1 \text{ hr}} =$$

$$\frac{11{,}562 \times 1 \times 1 \times 288}{1 \times 60 \times 60 \times 1} \text{ dollars} =$$

$$\frac{3{,}329{,}856}{3{,}600} \text{ dollars} =$$

924.96 dollars

The step-by-step approach is always the same, we keep cancelling labels until we get to the label we want. In this example, "seconds", "minutes", and "hours" all cancelled, leaving us with "dollars".

Note:

Using a calculator, we can:

1. *Enter 11,562*
2. *Divide by 60*
3. *Divide by 60 (again)*
4. *Multiply by 288.*

Dividing by 60 is the same as multiplying by 1/60

In this way, we can do the arithmetic without having to "reset" the calculator.

8.

This is a common problem we often face. We know an amount but have a different sized measuring device. In this problem we know the number of cups but all we have is a tablespoon. The key point is that we know how to convert cups to tablespoons.

For example:

$$2\tfrac{1}{2} \text{ cups} = 2\tfrac{1}{2} \times 1 \text{ cup}$$

$$= 2\tfrac{1}{2} \times 8 \text{ ounces}$$

$$= \frac{5}{2} \times 8 \text{ ounces}$$

$$= 20 \text{ ounces}$$

$$= \frac{20 \text{ ounces}}{1} \times \frac{1 \text{ tablespoon}}{2 \text{ ounces}}$$

$$= 10 \text{ tablespoons}$$

We're given this conversion factor in the exercise.

And here's where we use the fact that there are 2 ounces per tablespoon.

8. (cont)

If you're still having trouble with the conversions, here is a step-by-step process that might help.

Step 1:

Read the problem carefully and rewrite it in a fill-in-the-blank form. *In this exercise we want to convert cups to tablespoons (tbsp), so we have:*

$$2\tfrac{1}{2} \text{ cups} = \_\_\_\_\_ \text{ tbsps}$$

You may write this as 2.5 if you're using a calculator.

Step 2:

Write down all of the given relationships. *In this exercise, the ones we have are:*

8 ounces = 1 cup

2 ounces = 1 tablespoon

Here we're using "=" more as a synonym for "per"

Step 3:

Return to Step 1 and use Step 2 to help you get from the given label to the missing label. *In this exercice the given label is "cups". From Step 2, we see that we can relate "cups" to ounces (8 ounces = 1 cup) and we can then relate ounces to tablespoons (2 ounces = 1 tablespoon).*

Step 4:

Translate the information into the common fraction form:

$$2.5 \text{ cups} = \frac{2.5 \text{ cups}}{1} \times \frac{8 \text{ ounces}}{1 \text{ cup}} \times \frac{1 \text{ tbsp}}{2 \text{ ounces}} =$$

Remember to put the conversion factor next to the correct label.

8. (concluded)

So:

$$2.5 \text{ cups} = \frac{2.5 \times \overset{4}{\cancel{8}} \times 1}{1 \times 1 \times \underset{1}{\cancel{2}}} \text{ tbsps}$$

$$= 10 \text{ tablespoons.}$$

If you'd like to practice this procedure, review any of the previous 7 exercises that gave you trouble, using the "new" method.

9.

Here we have another measurement problem using constant rates, but the wording has been changed to help emphasize the need to read the problem correctly.

(a) We want to put 48 kilograms into 1 pound bags; so in terms of fill in the blank we have:

48 kilograms = _____ (1 pound) bags.

The relationships are:

454 grams = 1 pound

1,000 grams = 1 kilogram.

This was given earlier but not mentioned here. Either you have to remember this or else look it up.

So we'll convert the given label "kilograms" to "grams" and then "grams" to "pounds".

$$\frac{48 \text{ kg}}{1} \times \frac{1{,}000 \text{ gm}}{1 \text{ kg}} \times \frac{1 \text{ pound}}{454 \text{ gm}} =$$

$$\frac{48{,}000}{454} \text{ pounds} =$$

$$105.72687 \text{ pounds}$$

I used the calculator to get this answer; but as we shall see in part (b), this form is not acceptable as it stands to give us the number of ounces that remain after the bags are filled.

which is more than 105 pounds but less than 106 pounds. Hence 105 1-pound bags are filled; and part of the 106th bag is filled. What part this is becomes the subject of part (b).

9(b)

In part (a) we saw that we had 105.72687 pounds. The problem is that we are more familiar with ounces than with decimal fraction parts of a pound.

The key here is to remember that 0.72687 pounds means:

0.72687 is about 3/4; so we know that the answer has to be about 12 ounces.

0.72687 X 1 pound =

0.72687 X 16 ounces =

11.62992 ounces $\doteq$

12 ounces

We have to know that there are 16 ounces per pound

That is, 48 kilograms to the nearest ounce is 105 pounds 12 ounces.

Notice how much more convenient it is to say "12 ounces" than to say "0.72687 pounds"

10.

Again we have to read the problem and see what we're asked to find. In terms of fill in the blank, the problem is:

1 pound 10 ounces = ______ pinches

and if we recall that there are 16 ounces per pound, the problem becomes:

26 ounces = _____ pinches (1)

1 pound 10 ounces is the same as 16 ounces + 10 ounces

The problem also tells us the following relationships:

7,000 grains = 1 pound

1 pinch = 20 grains

Of course we can also rewrite 1 pound 10 ounces in terms of pounds; namely: 1 10/16 lbs or 26/16 or 13/8 pound; and then work with the fill in the blank:

$$\frac{13}{8} \text{ pounds} = \_\_\_\_ \text{ pinches}$$

Since ounces aren't mentioned in these relationships we supply the fact that there are 16 ounces per pound. We then go from

10.

ounces to pounds, from pounds to grains, and from grains to pinches. Using (1) and the other given relationships we get:

$$26 \text{ ounces} = \frac{26 \text{ ounces}}{1} \times \frac{1 \text{ pound}}{16 \text{ ounces}} \times \frac{7{,}000 \text{ grains}}{1 \text{ pound}} \times \frac{1 \text{ pinch}}{20 \text{ grains}}$$

$$= \frac{26 \times 1 \times 7{,}000 \times 1}{1 \times 16 \times 1 \times 20} \text{ pinches}$$

$$= \frac{182{,}000}{320} \text{ pinches}$$

$$= 568.75 \text{ pinches}$$

$$\doteq 569 \text{ pinches}$$

By this time you should be solving these exercises using any of the several methods we've discussed in this solution set. In a given problem, pick the method you're most comfortab with.

In concluding this solution set, keep in mind that while there are virtually an endless number of ways in which constant rates occur in measurement problems, we can always solve them by the methods that were presented in this module. Nevertheless we'll give more illustrations in the Self-Test for Module 11.

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Step 7:

Do Self-Test 10, Form B on the next page.

Self-Test 10, Form B

ANSWERS:

1. To the nearest foot, how many yards are there in 120 meters? 1. ______

2. To the nearest centimeter, how many meters are there in 120 yards? 2. ______

3. At a cost of $255 per hectare, how much will 63 acres cost? Round off your answer to the nearest dollar. 3. ______

4. At a cost of $255 per acre, to the nearest dollar, what is the cost of 63 hectares? 4. ______

5. At a rate of $0.0097 per gram, what is the cost, to the nearest cent. of: 5. (a) ______ (b) ______

   (a) 2 pounds?

   (b) 2 metric tons?

6. At a rate of 51 miles per hour, how far will you travel in 4 hours and 17 minutes? Write your answer to the nearest tenth of a mile? 6. ______

7. Access time to a computer costs $252 per hour, where the time is measured to the nearest second. At this rate, what will it cost you to use the computer for 2 hours, 11 minutes, and 37 seconds? 7. ______

8. A tablespoon is 2 ounces and a cup is 8 ounces. A recipe calls for 3 1/4 cups of flour. You do not have a measuring cup but you do have a tablespoon. How many tablespoons of flour should you use in this recipe? 8. ______

9. 53 kilograms of tobacco are packed into 2-pound bags. 9. (a) ______ (b) ______

   (a) How many of these bags are filled by the tobacco?

   (b) To the nearest ounce, how many ounces of tobacco are left after the 2-pound bags are filled?

10. There are 7,000 grains in 1 pound. If a "pinch" of salt is 20 grains; to the nearest whole number, how many pinches are there in a 2 pound 5 ounce box of salt? 10. ______

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 10, Form B

1. *131 yards, 1 yard*
2. *109 meters, 73 centimeters*
3. *$6,501*
4. *$39,697*
5. *(a) $8.81   (b) $19,400*
6. *218.5 miles*
7. *$552.79*
8. *13 tbsps*
9. *(a) 58   (b) 6 ounces*
10. *809*

****

If you did each problem in Self-Test 10, Form B correctly, you may, if you wish, proceed to the next module. Otherwise continue with Step 8.

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Step 8:

View the solutions for Self-Test 10, Form B on Videotape Lecture 10S. Pay special attention to the solutions of those problems for which you failed to get the correct answers. Feel free to rewind the tape at any time to restudy the problems that gave you difficulty.

Step 9:

Do Self-Test 10, Form C on the next page.

Self-Test 10, Form C

ANSWERS:

1. To the nearest foot, how many yards are there in 220 meters? — 1. ____________

2. To the nearest centimeter, how many meters are there in 220 yards? — 2. ____________

3. At a cost of $155 per hectare, how much will 37 acres cost? Round off your answer to the nearest dollar. — 3. ____________

4. At a cost of $155 per acre; to the nearest dollar, what is the cost of 37 hectares? — 4. ____________

5. At a rate of $0.0083 per gram, what is the cost, to the nearest cent, of:

   (a) 3 pounds? — 5. (a) ____________

   (b) 3 metric tons? — (b) ____________

6. At a rate of 47 miles per hour, how far will you travel in 2 hours and 29 minutes? Round off your answer to the nearest tenth of a mile. — 6. ____________

7. Access time to a computer costs $216 per hour, where the time is measured to the nearest second. At this rate, what will it cost to use this computer for 2 hours, 13 minutes, and 47 seconds? — 7. ____________

8. A tablespoon is 2 ounces and a cup is 8 ounces. A recipe calls for 4 3/4 cups of milk. You have a tablespoon but no measuring cup. How many tablespoons of milk should you use in this recipe? — 8. ____________

9. 61 kilograms of tobacco are packed into 3-pound bags.

   (a) How many of these bags are filled by the tobacco? — 9. (a) ____________

   (b) To the nearest ounce, how many ounces are left after the 3-pound bags are filled? — (b) ____________

10. There are 7,000 grains in a pound. If a "pinch" of salt is 20 grains; to the nearest whole number, how many pinches are there in a 1 pound 11 ounce box of salt? — 10. ____________

(ANSWERS ARE ON THE NEXT PAGE)

Answers for Self-Test 10, Form C

1. *240 yards 2 feet*

2. *201 meters 17 centimeters*

3. *$2,321*

4. *$14,171*

5. *(a) $11.30 (b) $24,900*

6. *116.7 miles*

7. *$481.62*

8. *19 tbsps*

9. *(a) 44 (b) 13 ounces*

10. *591*

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THIS CONCLUDES OUR STUDY GUIDE PRESENTATION FOR MODULE 10.

HOPEFULLY, YOU WILL NOW FEEL READY TO BEGIN MODULE 11.

HOWEVER, IF YOU STILL FEEL UNCERTAIN OF THE MATERIAL IN THIS MODULE, YOU SHOULD CONSULT WITH A TEACHER, A FRIEND, OR A FELLOW-STUDENT FOR ADDITIONAL REINFORCEMENT.

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